

## ULTRACAPACITOR SIZING

Tecate Group offers a large selection of ultracapacitor cells and modules for various applications. In order to size the appropriate ultracapacitor cell for any application, we will need to determine the system variables needed. Using this information we can calculate the appropriate size and number of cells needed.

In order to get a complete solution, the following parameters will need to be defined:

- Maximum Charged Voltage ( $V_{max}$ ), if different from Working Voltage then also ( $V_w$ )
- Minimum Voltage ( $V_{min}$ )
- Required Power (W) or Current (I)
- Duration of Discharge ( $t_d$ )
- Duty Cycle
- Required Life
- Average Operating Temperature

The last three parameters are used to determine the life degradation factor to use. This is not discussed in this application note but is a consideration to be taken by user. In order to know the appropriate size and also the number of cells required one needs to perform some simple sizing exercise. Most applications can be categorized into two categories: *constant current applications* or *constant power applications*. We will examine each one separately in this application note.

During the discharge cycle of an ultracapacitor there are two parameters to consider. The drop in voltage due to internal resistance, and the drop in voltage due to capacitance, as shown in Figure 1.

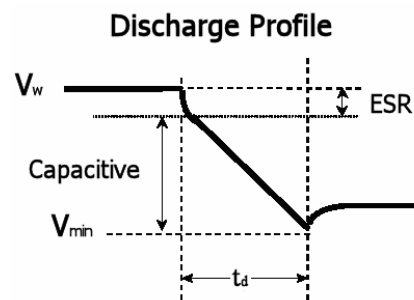


Figure 1: Discharge Curve

As can be seen above during a discharge cycle the initial drop in voltage is due to the Equivalent Series Resistance of the part (ESR). The amount of drop is a function of the ESR and discharge current as indicated by the equation below:

$$dV_{ESR} = I * ESR \quad \text{Equation 1}$$

After the initial instantaneous drop due to ESR, the capacitor will discharge according to its capacitance and discharge current as indicated in equation 2:

$$dV_{cap} = I * t_d / C \quad \text{Equation 2}$$

By placing these two equations together the total voltage drop can be calculated per equation 3:

$$dV_{\text{Total}} = I * t_d / C + I * \text{ESR} \quad \text{Equation 3}$$

A brief overview of the variables in above equation:

$dV_{\text{Total}}$  = The drop in voltage when the capacitor is discharged. This is the difference between the  $V_w$  and  $V_{\text{min}}$  as indicated on Figure 1. As can be seen in equation 3 this is the sum of the resistance and capacitance drop. *Note: Allowing a larger  $dV$  will reduce the capacitance size used. Typically by allowing the capacitor to drop to  $1/2 V_w$ , 75% of the capacitor energy is discharged.*

$I$  = Current in Amps used to discharge the capacitors. For equation 3 we assume this to be a constant current discharge.

$t_d$  = Duration in Seconds to discharge the capacitor between  $V_w$  and  $V_{\text{min}}$ .

$C$  = the total capacitance of the ultracapacitor. If a single cell is used, then it is the cell capacitance. If multiple cells are used the equivalent capacitance is based on the number of capacitors in series or parallel. For capacitors in series the capacitance is additive at  $1/C$ . For capacitors in parallel the capacitance is additive.

$$C_{\text{Total}} = C_{\text{cell}} * (\# \text{ parallel} / \# \text{ series}) \quad \text{Equation 4}$$

$\text{ESR}$  = the total resistance of the ultracapacitor. If single cell is used then it is the cell resistance. If multiple cells are used the equivalent resistance is based on the number of capacitors in series or parallel. For capacitors in series the resistance is additive. For capacitors in parallel the resistance is additive at  $1/\text{resistance}$ .

$$\text{ESR}_{\text{Total}} = \text{ESR}_{\text{cell}} * (\# \text{ series} / \# \text{ parallel}) \quad \text{Equation 5}$$

## Constant Current Application

In the example below we will look at an application where the load requirement is a constant current.

### Example 1:

Let's assume we have a requirement as follows:

$$\begin{aligned} V_{\text{max}} &= 15\text{V} \\ V_{\text{min}} &= 9\text{V} \\ I &= 4 \text{ A} \\ t_d &= 5 \text{ sec} \end{aligned}$$

Using the information above we can use two methods to try and find the appropriate capacitor size.

#### Method 1:

We can ignore the ESR effect on the voltage drop to get an estimate on the capacitance and then resolve Equation 3 to see if the size picked is appropriate with the ESR effect, and if not, to increase to the next cell size.

So by taking out the ESR portion of equation 3 and solving for  $C$  we get:

$$C = I * t_d / dV \quad \text{Equation 6}$$

By substituting the above parameters we get:

$$C = 4 \cdot 5 / (15 - 9) = 3.33F$$

Note that this is the total capacitance at 15V. Since each ultracapacitor cell is typically rated at 2.7V, then we divide  $V_{max}$  by 2.7 and round up:

$$15 / 2.7 = 5.5, \text{ so } 6 \text{ cells in series.}$$

Using equation 4 and assuming one parallel set then we estimate the cell needed will be in the range of:

$$C_{cell} = C_{Total} \cdot \# \text{ in series} = 3.3 \cdot 6 = 19.8F$$

Looking at the product offering we see the closest size is a 22F cell with an ESR value of 45 mohm. Taking the capacitance and ESR value of this cell we can plug it into equation 3 to see if the total voltage drop is within the application limit of 6V (note that we need to calculate the system C and ESR for 6 cells in series):

$$dV_{Total} = (4 \cdot 5 / (22 / 6)) + (4 \cdot (.045 \cdot 6)) = 5.46 + 1.08 = 6.54V$$

As can be seen the above voltage drop is more than the 6V allowed. Therefore we need to move to the next size up cell and redo the calculation. Also please note the values used are the initial specifications. To take into consideration end of life degradation one needs to apply the degradation factors to the capacitance and resistance numbers.

#### *Method 2:*

For the same parameters as above we can solve for the cell value using the RC time constant. The RC time constant of an ultracapacitor is the product of its capacitance value and resistance value. For the Tecate product TPL series we can use 0.8 seconds as an average.

$$\text{Since } R \cdot C = 0.8 \text{ seconds, then } R = 0.8 / C$$

By substituting the above in equation 3 we get the following:

$$dV_{Total} = I \cdot t_d / C + I \cdot 0.8 / C$$

Solving for C, we get:

$$C = I / dV_{Total} \cdot (t_d + 0.8)$$

By substituting the value give ( $dV=6$ ,  $I=4$  and  $t_d=5$ ) we solve the above equation for C:

$$C = 3.86F$$

Please note this is the stack capacitance and we need to solve for the cell capacitance. Based on the  $V_{max}$  of 15V we determined we need 6 cells in series. So using equation 4 we solve for the cell capacitance as follows:

$$C_{cell} = C_{Total} \cdot \# \text{ series}$$
$$C_{cell} = 3.86 \cdot 6 = 23.16F$$

We will then use the data sheet to round this up to the closest cell available, which in this case is 25F.

## Constant Power Application

In the example below we look at an application where the load is on a constant power discharge.

### Example 2:

Let's assume we have a requirement as follow:

$$\begin{aligned}V_{\max} &= 15\text{V} \\V_{\min} &= 9\text{V} \\ \text{Power (P)} &= 60\text{W} \\t_d &= 5 \text{ sec}\end{aligned}$$

Using the information above we can use two methods to try and find the appropriate capacitor size.

#### Method 1:

A simple method will be to calculate an average current based on the above parameters and then apply the constant current sizing method to calculate the most appropriate cell:

$$\begin{aligned}I_{\max} &= \text{Power}/V_{\min} = 60/9 = 6.67 \text{ Amps} \\I_{\min} &= \text{Power}/V_{\max} = 60/15 = 4 \text{ Amps} \\I_{\text{avg}} &= (6.67 + 4)/2 = 5.34 \text{ Amps}\end{aligned}$$

Using the method 1 sizing of the constant current application mentioned earlier we get a cell value of 26.7F. Rounding it up to the next cell value we find out the 30F cell is the appropriate cell for this application. *Please note we typically recommend oversizing the cell by 20-30% to accommodate any degradation over time.*

#### Method 2:

We can also calculate the appropriate cell size based on the energy needed. This method works well for low power application where the loss due to ESR is minimal.

$$\text{Energy Needed} = 60\text{W} * 5 \text{ sec} = 300 \text{ Joules}$$

The equation calculating the energy stored in a capacitor is:

$$E \text{ (joules)} = 0.5 * C (V_{\max}^2 - V_{\min}^2) \quad \text{Equation 8}$$

Substituting the values in equation 8 and solving for C we get:

$$\begin{aligned}300 &= 0.5 * C (225 - 81) \\C &= 4.17 \text{ F}\end{aligned}$$

Since we have 6 cells in series the cell capacitance needed is:

$$4.17 * 6 = 25.02 \text{ F}$$

Note that the above method did not take into account any losses due to ESR, thus resulting in a slightly smaller cell value.